POST-CRISIS METHODS FOR BOOTSTRAPPING THE YIELD CURVE

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Abstract

Traditional swap curve construction does not treat the front end of the curve in a very sensible way, as it ignores market expectations of MPC rate announcements. It uses LIBOR as a proxy for such expectations. In Europe and the US (and everywhere but SA, it seems) this problem came to a head with LIBOR becoming a fiction. Soon thereafter overnight interest rate swaps became very liquid and became the accepted instruments for use in the short part of the curve; use of these instruments continues even with the return of LIBOR to reality. We talk about including OISs and plain vanilla swaps in a bootstrap in a way which is consistent with standard bootstrap-interpolation algorithms, such as the one of Pat Hagan and the author.

1. Overnight indexed swaps

Some acronyms to begin:

- Sterling Overnight Index Average (SONIA). SONIA is the weighted average rate to four decimal places of all unsecured sterling overnight cash transactions brokered in London.
- Euro OverNight Index Average (EONIA) is an effective overnight rate computed as a weighted average of all overnight unsecured lending transactions in the interbank market. It has been initiated within the euro area by the contributing panel banks.
- Federal Funds Effective rate is the weighted average rate for overnight lending of deposits at the Federal Reserve.

An OIS is a swap of fixed interest for floating accrued interest, where the floating rate is an overnight rate, and capitalisation occurs daily throughout the period of the swap. For OISs of tenor up to a year, there is a single payment at the end of the swap (usually on the day after the maturity of the swap), representing the difference in interest on the two legs of the swap. If the swap has maturity greater than 1y then cashflows are annual, with an odd first period (stub) if required.

- The fixed leg can be considered much like a synthetic deposit, and is quoted in the market as a yield that is applied over the tenor of the swap.
- The floating leg, on the other hand, compounds principle plus interest on a daily basis. The daily fix is the weighted-average of overnight cash deposits traded that day. In the event of a weekend or other public holiday, the compounding is done on a simple basis over that period. (That is, Friday rate count 3 times, etc.)

The product is net settled.

Market expectation is that the daily forward rates will be equal to the ruling overnight central bank policy rate. Thus, a position in an OIS encompasses a view on future policy rates.

Note that the explanation on Wikipedia http://en.wikipedia.org/wiki/Overnight_indexed_swap and the Credit Suisse source it refers to http://www.acisuisse.ch/docs/dokumente/OIS_Note_CSFB_Zurich.pdf are grossly erroneous. They refer to 'interest accrued through geometric

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averaging of the floating index rate' which is clearly nonsense - this quantity is not a financial variable, and can be 0 without the actual accrual of interest being 0.

For a valid presentation see 'The short end of the curve' by Chris Pulman: http://www.wilmott.com/attachments/The_Short_End_of_the_Curve_41.pdf

To find market quotes for OIS rates one can simply refer to an index which is, like an interbank rate, determined by a panel of banks. This is discussed in http://www.aciforex.com/docs/markettopics/EONIA_SWAP_INDEX_BrochureV2-2008-01044-01-E.pdf; in this document an OIS is properly defined and examples are given.

Suppose the current policy rate is r_0.

Suppose there are central bank meeting announcements at times $t_1, t_2, \ldots$ [1] and consequential market expected policy rates $r_1, r_2, \ldots$. Thus, r_i applies for the time period $[t_i, t_{i+1})$. Our intention is to discover these market expected rates from the OIS's trading in the market.

Suppose there are OIS swap expiry dates $E_1, E_2, \ldots$ - so for the j^{th} swap overnight compounding occurs on $[t_0, E_j)$ - with swap rates $R_1, R_2, \ldots$.

We might hope that $t_j < E_j < t_{j+1}$ for $1 \leq j$. However, there are at least two reasons why this might not be the case:

- Swaps are only trading for every month in the first year, say, and subsequently less frequency. However, the central bank meeting dates are known for at least two years.
- Central bank meeting dates are not set according to the modified following rule, whereas swap expiries are. Thus, between two successive meeting dates there could be two swap maturity dates: the first very soon after the first meeting and the second just before the second meeting. For example, on 6 Oct 09 the 3 month date is 6 Jan 10 and the 4 month date is 8 Feb 10. However, there are meetings ending on both of 7 Jan 10 and 4 Feb 10.

  In this case, if swaps for both maturity dates are trading, then we have to select one and reject the other in a bootstrap. Alternatively, the bootstrap application could specify if arbitrage is available - there will almost surely be mathematical arbitrage, but not if bid-offer spreads are taken into account. To be decided.

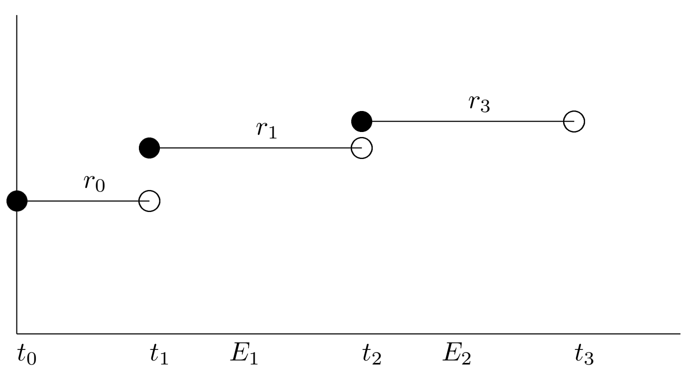

FIGURE 1. Term structure of MPC meetings and swap tenors

Finally, suppose the day count basis is D (typically, $D = 365$ or $D = 360$).

[1]The last (typically second) day of the scheduled MPC meeting.

1.1. **Overnight indexed swap payment calculation - tenors less than one year.** Consider the OIS with index M. Then

$$\left[1 + R_M \frac{t_M - t_0}{D}\right] = C(t_0, t_M) \tag{1}$$

as any payment delay figures equally on both sides.

1.2. **Overnight indexed swap payment calculation - tenors greater than one year.** Any OIS of tenor greater than one year has annual settlements, with the stub being at the front.

As a first example, suppose we have an OIS of maturity M months where $12 < M \leq 24$.

We ignore payment delays (typically one day). Then the equation of value is that

$$\begin{aligned} &R_M \frac{t_{M-12} - t_0}{D} Z(t_0, t_{M-12}) + R_M \frac{t_M - t_{M-12}}{D} Z(t_0, t_M) \\ &= (C(t_0, t_{M-12}) - 1) Z(0, t_{M-12}) + (C(t_0; t_{M-12}, t_M) - 1) Z(t_0, t_M) \end{aligned}$$

and so

$$Z(t_0, t_M) = \frac{1 - \left[R_M \frac{t_{M-12} - t_0}{D}\right] Z(t_0, t_{M-12})}{1 + R_M \frac{t_M - t_{M-12}}{D}} \tag{2}$$

In general, suppose the stub period is of length α_1 and subsequent annual periods are $\alpha_2, \ldots, \alpha_n$. Suppose the fixed rate is R. Then the fixed payments have value $R \sum_{i=1}^{n} Z(0, t_i)$ and the floating payments have value $1 - Z(0, t_n)$. Thus we get

$$Z(t, t_n) = \frac{1 - R \sum_{j=1}^{n-1} \alpha_j Z(t, t_j)}{1 + R_n \alpha_n} \tag{3}$$

A solution is found as in the iterative (fixed point) technique for ordinary swap curves seen in Hagan and West [2006], Hagan and West [2008].

1.3. **Vanilla interest rate swaps.** Included as for vanilla swap curve bootstraps.

1.4. **Flattening of the yield curve between MPC meeting dates.** By choice we have bootstrapped a yield curve where the instantaneous forward rates might be continuous (as is the case for monotone convex interpolation, and most mathematical splines, for example). What we now want to do is preserve the general shape of the curve we have created, but adjust for MPC dates.

The overnight rate of interest should be constant between any two adjacent MPC meeting dates. Thus, our aim is now to 'flatten' the forward rates on every interval $[t_i, t_{i+1})$.

For $p > 0$, let $N(s, t, p)$ be the number of occurrences of p many calender days from the one business day to the next in the period $[s, t)$. Then the required overnight rate of interest r for the period $[t_{i-1}, t_i)$ is the solution to

$$C(t_0; t_{i-1}, t_i) = \prod_{p>0} \left(1 + \frac{pr}{D}\right)^{N(t_{i-1}, t_i, p)} := f(r) \tag{4}$$

How do we solve this equation? Suppose $C(t_0; s, t) = y$. The right hand side is very nearly equal to $\left(1 + \frac{r}{D}\right)^{t-s}$, so we have as the first estimate

$$r^1 = D\left[y^{\frac{1}{t-s}} - 1\right]. \tag{5}$$

A recursion using Newton's method is

$$r^{m+1} = r^m - \frac{1 - \frac{y}{f(r^m)}}{\sum_{p>0} \frac{pN(s,t,p)}{D+pr^m}} \tag{6}$$

Suppose then we have solved for r. Suppose that $C(0,t)$ has been determined for $t \in [s,t)$ then

$$C(0, \mathrm{nbd}(t)) = C(0,t)\left(1 + \frac{\mathrm{nbd}(t) - t}{D} r\right) \tag{7}$$

and we continue recursively. This then is the final yield curve. It is has flat segments as long as there are MPC dates, and has all the desired smoothness properties thereafter.

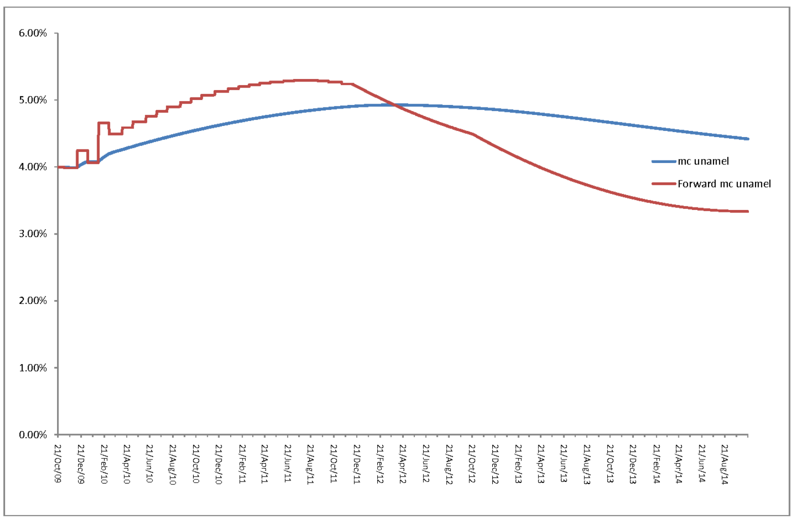

FIGURE 2. A curve bootstrapped with OISs and a vanilla swap

2. Central bank meeting date

Monetary policy comittee meeting dates can be found on their respective websites:

- ECB: http://www.ecb.int/events/calendar/mgcgc/html/index.en.html
- BOE: http://www.bankofengland.co.uk/monetarypolicy/decisions.htm
- FOMC: http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm

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