MSc in Mathematics of Finance, 2008

South African Financial Markets

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(1) You have a choice of 2 investments:
   (i) R100 000 invested at 12.60% NACS for 1 year
   (ii) R100 000 invested at 12.50% NACQ for 1 year.
   Which one do you take and why? (Explain Fully)

(2) On 17-Jun-08 you sell a 1 Million, 3 month JIBAR instrument. At maturity you receive R1,030,246.58. What was the JIBAR rate?

(3) (a) If I have an x% NACA rate, what is the general formula for y, the equivalent NACD rate in i.t.o x?
   (b) Now generalise this to convert from **any** given rate, r_1 (compounding frequency d_1) to another rate r_2 (compounding frequency d_2).
   (c) Now construct an efficient algorithm that converts from or to simple, NAC* or NACC. (Hint: given the input rate, find what the capitalisation factor is (for a year or for the period specified), if the rate is simple. Then, using this capitalisation factor, find the output rate.)

(4) On 17-Jun-08 you are given that the 3-month JIBAR rate is 12.00% and the 3x6-FRA rate is 12.10%. What is the 6 month JIBAR rate?

(5) If the 4 year rate is 9.00% NACM and the 6 year rate is 9.20% NACA, what is the 2 year forward rate (NACA) for 4 years time?

(6) Suppose on 22-Jan-08 the 3 month JIBAR rate is 10.20% and the 6 month JIBAR rate is 10.25%. Show that the fair 3x6 FRA rate is 10.0446%.

(7) Suppose on 3-Jan-08 a 6x9 FRA was traded at a rate of 12.00%. It is now 3-Apr-08. The NACC yield curve has the following functional form: $r(t,T) = 11.50\% + 0.01(T-t) - 0.0001(T-t)^2$ where time as usual is measured in years. Find the MtM of the receive fixed, pay floating position in the FRA.

   Note that the 6x9 FRA has now become a 3x6 FRA. However, the rate 12.00% is not the rate that the market would agree on for a new 3x6 FRA. Using the yield curve, what would be the fair rate for a newly agreed 3x6 FRA? (There are two ways of doing this: from scratch, or by using the information already calculated.)

(8) Relevant Bond data for a selection of SA bonds is provided. Given Inputs:
   - Bond Name
   - Settlement Date
   - YTM

   build a Bond Calculator to:

(a) Bring in (given the above inputs) all the relevant static data for that particular bond - such as maturity, coupon dates etc. (This will be done by means of an excel vlookup table.)

(b) Calculate:

- The LCD, BCD and NCD as efficiently as possible. A good trick is to break every date you consider into a cell containing the year, a cell containing the month, and a cell containing the day. The logic can then revolve around manipulation with these numbers: manipulation with numbers is so much easier than manipulation with dates. To recover the corresponding date, you use the DATE function.
- The Bond Allin Price, both rounded and unrounded.
- The Clean Price, both rounded and unrounded.
- The Accrued interest amount, both rounded and unrounded.
- the delta and rand per point;
- the duration and modified duration;
- the gamma and convexity.

This must be done entirely in excel. No macros, no vba.

(9) Suppose V is the value of a set of cash flows (for example, a bond) which has payments $c_1,\ c_2,\ \ldots,\ c_n$ at times $t_1,\ t_2,\ \ldots,\ t_n$. Note that $V = \sum_i c_i Z(0,t_i) := \sum_i V_i$ where $Z(\cdot)$ is for example calculated off of a yield curve. If the bond has a NACS yield to maturity y, then we take $Z(0,t_i) = (1+\frac{y}{2})^{-2t_i}$.

Let $w_i = \frac{V_i}{V}$ be the proportion that the i^{th} component makes up of the total value of the bond. We define (Macauley) duration and modified duration as

$$D = \sum_i t_i w_i = \frac{\sum_i t_i V_i}{\sum_i V_i}$$

$$D_m = \frac{D}{1+\frac{y}{2}}$$

Note that the units of Macauley duration and modified duration are years.

Show that these quantities can in fact be calculated as in the notes, as functions of the delta of the bond.

(10) Suppose on 3-Mar-08 I have the JIBAR and FRA data given in the table.

jibar	fra	fra	fra	fra	fra	fra	fra
3m	3v6	6v9	9v12	12v15	15v18	18v21	21v24
11.33%	11.43%	11.41%	11.29%	11.04%	10.74%	10.74%	10.74%

(a) Bootstrap a the rates for every 3 month interval out to 24 months.

(b) What is the fair swap rate for a 24 month swap?

(c) Suppose we decide to use raw interpolation to find rates at dates which are not are not node points on my yield curve. Write a vba function to find the NACC rate at any non-node point.

(d) Suppose on 12-Feb-08 the 3m JIBAR rate was 11.20% and the 6v9 FRA rate was 11.40%. I entered into a pay fixed 6v9 FRA. What is the MtM of the FRA now (on 3-Mar-08)?

(e) Suppose on 12-Feb-08 I entered into a 1 year swap, paying a fixed rate of 11.30%. What is the MtM of the swap now (on 3-Mar-08)?

(11) Suppose on 3-Apr-08 my yield curve is 12.00% NACC for a term of zero, and increases by one-tenth of a basis point every calendar day into the future (relevant for this question).

- Find the fair rate for vanilla swaps with expiry at 6m, 9m, 12m, etc. to 20y.
- What can you observe about the trend in the fair swap rates? What is the reason for this?
- If I am paying fixed, receiving floating, in what periods do I expect to be receiving payments and in what periods do I expect to make payments?

(12) (UCT exam 2004) Suppose we are given the inputs to the swap curve as follows:

Date	31-Mar-05	
Swap Curve Inputs		
Data	Type	Rate
1 day	on	7.6400%
1 month	jibar	7.2210%
3 month	jibar	7.4040%
3x6	fra	7.2500%
6x9	fra	7.2900%
9x12	fra	7.4000%
12x15	fra	7.8500%
15x18	fra	8.1900%
2y	swap	7.8300%
3y	swap	8.3200%
4y	swap	8.6200%
5y	swap	8.8700%

Bootstrap the yield curve out to 5 years using the method discussed in class. Report the rates for every 3 month point following the valuation date out for 5 years.

To determine these three month points, use the modfol function provided. Use raw interpolation on the NACC rates, for which it is recommended that you write your own linear interpolation function in vba.

(13) Suppose that an index consists of the following shares:

Share	Price	ffSISS
AMS	22.00	6 722 223
BOC	4.52	40 000 000
MMD	3.20	22 331 700
ZZZ	0.45	17 010 000
ABC	7.20	10 000 555

The index level at close of business today is 8005.22. Two events now occur:

(a) AMS has a 5 for 1 share split.

(b) ZZZ has deteriorated in terms of market capitalisation and is removed from the index. It is replaced by PAR which has a closing price of 12.02 and a ffSISS of 10 000 270.

Calculate the old and the new basing constant (k Factor).

Answer: 1602.7576 and 1566.8818.

(14) Consider the recent history of dividends for the share, ABC:

LDT Date	Dividend Amount
18-Jun-07	4.50
18-Dec-07	3.75

Using an interest rate of 12% NACC throughout, and the dividend model discussed in class, calculate for 26 February 2008,

(a) the JSE quoted dividend yield
(b) the expected dividend yield for a 3 month period
(c) the expected dividend yield for a 6 month period
(d) the expected dividend yield for a 5 year period

The share price for ABC on 26 February 2008 was 122.00 and there were no corporate actions in the last year.

(15) (UCT exam 2007) Suppose share ABC has the dividend information as follows:

LDR	Pay	Classification	Amount
08-Sep-06	15-Sep-06	Occurred	2.00
09-Mar-07	16-Mar-07	Occurred	1.50
06-Jul-07	13-Jul-07	Special, declared	1.00
07-Sep-07	14-Sep-07	Forecast	2.50

Today is 3 June 2007 and the stock price is R150. Using a risk free rate of 8% throughout, use the model discussed in class to calculate the dividend yield for an option which expires on 31 Dec 2011.

(16) Suppose that we assume that, at least for moderate moves in stock price, the present value of dividends in the short term is unchanged. Derive a formula that shows, under these assumptions, how the dividend yield changes, as a function of the old dividend yield, the old stock price, and the new stock price.

Why is the phrase 'in the short term' important?

(17) Suppose a stock pays percentage dividends q_i at times t_i. What is the value today of the dividend payment at time t_2? How would you replicate this?

(18) Prove equations (6.14) and (6.15) by induction.

(19) Suppose a stock pays percentage dividends q_i at times t_i. What is the value today of the dividend payment at time t_2? How would you replicate this (no arbitrage replication)?

(20) Suppose we expect an investment, that costs P at time t_0 to deliver cashflows c_i at times t_i, for $i = 1,\ 2,\ \ldots,\ n$. Write down the algorithm that uses Newton Rhapson to calculate the IRR (NACC) of the investment.

(21) Build a loop in your bond spreadsheet that, given the all-in-price, gives the next guess at the ytm given a previous guess, using the Newton-Bailey method. (This method converges within 2 or 3 iterations, so by simply cutting and pasting the VALUE of the output cell into the input cell, and then recalculating, one can iterate. You can even write a macro that will do this copy-paste values.)

(22) A R1,000,000 nominal r153 carry is booked with BESA. The first leg settles on 5-May-08 at 12.00% and the second leg settles on 12-May-08. The carry is executed at 11.50% simple. What is the consideration and the yield to maturity of the second leg?

(23) A loan is entered into from 3-Mar-08 maturing 17-Mar-08. The loan is for approximately R100,000,000 and interest to be charged is 11.50% simple. The deal is to securitised by a

carry transaction in the bond market with a carry of r153 bonds. The ytm on the first leg is 12.00%.

Describe in detail the deals that have to be entered into at the bond exchange, specifying the relevant settlement dates, the nominal of the carry, and ytm of both the buy and sell-back leg. (The nominal will be a whole lot ie. a whole number of R1,000,000's.)

(24) This example illustrates as discussed in class how one can speculate on bonds without ever owning any. Suppose you are bearish on bonds, that is, you believe that they are going to lose value (yields are going to increase). The following story describes what you could do.

On 21-May-08 you sell R1 million nominal of the r153 in the ordinary bond market. When settlement is due you have already dealt to carry the position for one week - you carry at 8.40% simple yield. When settlement is again due, you again have already dealt to carry again for another two weeks at 8.30% simple interest.

In order to close out the position you do an ordinary trade in the bond market at the appropriate time.

Set out a schedule of settlements, show the timing and quantum of all cash and scrip flows. You must take into consideration all weekends and public holidays in your timing considerations. The bids and offers of the r153 on all the business days in the period of interest are shown below:

	Bid	Offer
21-May-08	10.64%	10.62%
22-May-08	10.74%	10.72%
23-May-08	10.76%	10.74%
26-May-08	10.76%	10.74%
27-May-08	10.91%	10.89%
28-May-08	11.06%	11.04%
29-May-08	11.38%	11.36%
30-May-08	11.54%	11.52%
02-Jun-08	11.60%	11.58%
03-Jun-08	11.55%	11.53%
04-Jun-08	11.57%	11.55%
05-Jun-08	11.54%	11.52%
06-Jun-08	11.57%	11.55%
09-Jun-08	11.69%	11.67%
10-Jun-08	11.73%	11.71%
11-Jun-08	11.74%	11.72%
12-Jun-08	11.53%	11.51%
13-Jun-08	11.63%	11.61%
17-Jun-08	11.60%	11.58%
18-Jun-08	11.67%	11.65%
19-Jun-08	11.60%	11.58%
20-Jun-08	11.58%	11.56%

As we can see, yields have indeed gone up. Have you made money? How much?

(25) A r153 carry is entered into on 29-Feb-08 at a trade ytm of 12.00%, for expiry 16-Oct-08. The carry rate is a simple rate of 11.50%. Calculate the reference price of the carry, the second consideration and the second yield to maturity.

Now convert the quoted carry rate to a NACC rate and repeat the example.

For the big prize, explain carefully why the answers are different. Which is correct?

For a smaller prize, or to get yourself started, answer the following question: if I earn a certain simple rate for a period and then the same simple rate for the next period, have I earned that simple rate for the entire period?

(26) Describe how relative value plays can be implemented if there is a significant disparity between the bond curve and the swap curve. (Phrase your answer as follows: 'If the bond curve is much lower/higher than the swap curve, then I will go long/short a bond and pay fixed/floating in a swap. During the swap, I ..... From this I make a profit, because.....') Numerical calculations or examples are not required!

(27) Find the rounded all in price of the r189 for the trade date of the 30-May-08 with a ytm of 2.90%. The basing constant (base CPI) of the r189 is 95.683870988.[1]

(28) As discussed in class, build a spreadsheet to calculate and graph the volatility of a time series of financial prices/yields/rates. Use:

(a) Unweighted volatility calculation

(b) 'unweighted window' volatility with N=90

(c) EWMA volatility with $\lambda = 0.99$

(29) Write vba code to price all varieties of the vanilla option pricing formulae. Use the symbols and the general approach from the notes.

(30) Find the rand premium for a Dec08 futures put option with the following criteria:

- Valuation Date: 1-Jun-08
- Underlying: abt
- Strike: 100.00
- Volatility: 42.00%
- Futures price: 105.00

(31) You are managing a portfolio of futures and futures options. If you are short 255 Mar09 ALSI40 futures call options with a strike of 33000.00 and long 150 Mar09 ALSI40 futures what is the mark-to-market of your portfolio (in Rands) on 30-Jun-08? The futures price is 29000.00 and the volatility is 33.00%.

(32) (Wits exam 2005) Write a vba function to price SAFEX futures options. For the cumulative normal function, use the cumnorm6 function provided, calling this function from within vba.

Suppose I am long 10000 TOP40 futures call options for expiry March 2006 on 26 and 28 April 05, strike 12050 and short 5000 futures for the same expiry. By using the information on the SAFEX MtM sheets provided, calculate the MtM of my portfolio on 26 and 28 April 05, and hence determine the change in portfolio value between those two days.

Answer the following questions:

(a) What is the delta of the call option on the first day?

(b) Hence, say why the futures position is a delta hedge for the call position.

[1].... and relevant CPI data can be found at www.statssa.gov.za. You don't need to subscribe: the requisite data is on the front page.

(c) Nevertheless, if your working is correct, you will find that the p&l between the two days is nearly R7m. Why has that happened? Is it because the hedge is only approximate, because the gains are over two days rather than one, or what?

(33) (Wits exam 2005) Explain heuristically how volatility skews and smiles translate to fat or thin tails of the pdf of prices, when we move from an analysis of implied volatility skew to an analysis of the pdf.

Breeden and Litzenberger showed how this relationship could be made completely precise, by calculating the exact mathematical relationship between the volatility skew, the SAFEX option prices at those volatilities, and the probability density function for the futures level on the expiry date. Carefully state and prove their main result.

Suppose I have an opinion on what the expiry date pdf should look like. Explain how this could be translated into a skew for trading, and how I would exploit differences between my skew and the skew trading in the market.

(34) What is the price of a European OTC call option on a 1,000,000 nominal r153 Bond with the following details:

- Deal Date: 1-Jul-08
- Carry rate: 12.00%
- Discount rate: 11.60%
- Spot Yield: 11.50%
- Strike Yield: 11.25%
- Price Volatility: 12.00%
- Option Expiry: Nov09